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Waiting time distributions in financial markets

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Abstract. We study waiting time distributions for data representing two completely different financial markets that have dramatically different characteristics. The first are data for the Irish market during the 19th century over the period 1850 to 1854. A total of 10 stocks out of a database of 60 are examined. The second database is for Japanese yen currency fluctuations during the latter part of the 20th century (1989–1992). The Irish stock activity was recorded on a daily basis and activity was characterised by waiting times that varied from one day to a few months. The Japanese yen data was recorded every minute over 24 hour periods and the waiting times varied from a minute to a an hour or so. For both data sets, the waiting time distributions exhibit power law tails. The results for Irish daily data can be easily interpreted using the model of a continuous time random walk first proposed by Montroll and applied recently to some financial data by Mainardi, Scalas and colleagues. Yen data show a quite different behaviour. For large waiting times, the Irish data exhibit a cut off; the Yen data exhibit two humps that could arise as result of major trading centres in the World.

PACS. 02.50.Ey Stochastic processes – 02.60.Ed Interpolation; curve fitting – 05.45.Tp Time series analysis

1 Introduction

It has been well known at least for thirty years (Clark 1973 [1]) that in financial markets not only prices, but also the waiting times between two consecutive transactions vary in a stochastic manner.

Recent studies (Scalas, *et al.* [2,3]), of high frequency data waiting time distributions, show how the Continuous Time Random Walk (CTRW), introduced by Montroll and Weiss in 1965 [4], and Fractional Brownian Motion can reproduce some statistical properties of waiting times between consecutive transactions of BUND and BTP futures, traded at LIFFE in London.

In this paper we focus mainly on "low frequency" data taken in the Irish stock market between 1850 and 1854. In order to test the generality of our (and other) results we make a comparison with high frequency data taken in Japanese Yen Currency Market between 1989 and 1998.

2 A bit of history

During the 19th century, Irish stock activity was recorded on a daily basis and characterized by waiting times that varied from a day to some months.

Deals were done on a 'matched bargain basis' with members of the exchange (*i.e.*, stockbrokers) bringing buyers and sellers together in essentially the same way as is done today *via* electronic trading, say. The only difference is that there are many more buyers and sellers. In the 19th century one informed one's stockbroker what one wished to trade (say sell) a certain quantity and requested he get the best quote or do the deal subject to a price limit (here a minimum price). He maintained a ledger of such pending deals and sought to meet, on the stock exchange floor, other member of the exchange trying to do the opposite deal.

Recent studies [5] of different 19th century markets find that they were well integrated. Dublin traded international shares – it was not solely a regional market. The largest shares – banks and key railways – represented quality investments for UK investors and were also traded in London. Smaller shares would primarily have been of only local interest. We see a mix of local small investment and large ones integrated with London and hence the world stock markets. World trends are thus reflected in the Irish market for which, at the time, there were no exchange controls. (From 1801 to 1922 Ireland was part of UK.)

3 Theory summary

If we assume that both the logprice jumps $\xi_i = x(t_i) - x(t_{i-1})$ (where $x(t) = \log(\text{PRICE}(t))$) and the waiting times between two consecutive trades $\tau_i = t_i - t_{i-1}$ are

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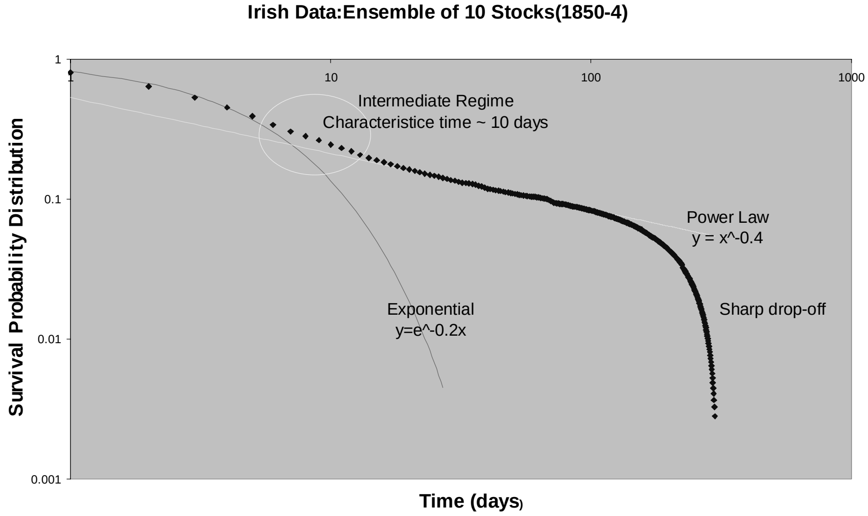

Fig. 1. Average Survival Time Probability Distribution function for Irish stock market data between 1850 and 1854. Fit parameters for M-L function: $\gamma = 0.025$ and $\beta = 0.4$.

i.i.d. stochastic variables characterized by the two probability density functions: $\lambda(\xi)$ and $\psi(\tau)$, it turns out that [2,3] the evolution equation for $p(x;t)$, the probability of having the log-price x at time t, can be written as:

$$\int_0^t \Phi(t-t')\frac{\partial}{\partial t'}p(x,t')\mathrm{d}t' = -p(x,t) + \int_{-\infty}^{+\infty} \lambda(x-x')p(x',t)\mathrm{d}x' \quad (1)$$

where Φ is a suitable memory kernel related in Laplace space to Ψ the probability that a given waiting interval is greater or equal to t (Survival Time Probability Distribution) – by:

$$\tilde{\Phi}(s) = \frac{\tilde{\Psi}(s)}{1 - s\tilde{\Psi}(s)}\,\cdot \quad (2)$$

The probability Ψ is given by:

$$\Psi(\tau) = \int_\tau^\infty \psi(t')\mathrm{d}t'. \quad (3)$$

The probability Ψ can be estimated from empirical data, and can thus be used to test hypotheses on the memory kernel Φ. In particular, if Φ exhibits a power-law time-decay:

$$\Phi \propto t^{-\beta} \quad (4)$$

with $0 < \beta < 1$ one can show that:

$$\Psi(t) \propto E_\beta\left(-t^\beta\right) \quad (5)$$

i.e.

$$\Psi(t)\begin{cases} = \displaystyle\sum_{n=0}^{\infty}(-1)^n \frac{t^{\beta n}}{\Gamma(1+\beta n)}, & t \geq 0 \\ \sim \displaystyle\frac{\sin(\beta\pi)}{\pi}\frac{\Gamma(\beta)}{t^\beta}, & t \to \infty \end{cases} \quad (6)$$

where $E_\beta(-t^\beta)$ is the Mittag-Leffler function of order β.

4 Empirical analysis

We analyse 10 stocks taken from Irish stock market in 19th century (5 railways and 5 banks). The data were recorded between the 1st of January 1850 and the 31st of December 1854. Waiting times range typically from a day to some months. In each time series we have 1826 days and a number of waiting times (*i.e.* of transactions) ranging between 140 and 160.

In order to minimize spurious effects related to single stocks and to capture the typical behaviour we work out the "average" Survival Time Probability Distribution (STPD) function.

The distribution in Figure 1 presents a sharp cut off for survival times bigger than 150 days. Below that time we can clearly identify two regions: the first one, ranging between 1 and 10 days presents an exponential behaviour (characteristic time 5 days), the second is well fitted by a power law function having an exponent $\beta = 0.4$. These two regions are very well fitted by a Mittag-Leffler function:

$$\Psi(t) = E_\beta\left[-(\gamma\tau)^\beta\right] \quad (7)$$

where $\gamma = 0.025$.

We estimate an uncertainty for the fit parameters of better than 10% and a $\tilde{\chi}^2 = 0.06$ (overlooking the cut off region).

We have performed the same analysis for Japanese Yen Currency. The data are taken between the 1989 and the 1998, the waiting times range from a minute to some hours. The STPD function (Fig. 2) shows in this case a very different behavior. Above 100 minutes, the distribution does not drop off sharply and presents two small humps or shoulders. The times associated with these shoulders may be associated with closing and opening times of the main currency markets in the World (New York, London, Tokyo).

For shorter times we no longer have a region that can be fitted easily with an exponential function – or at least

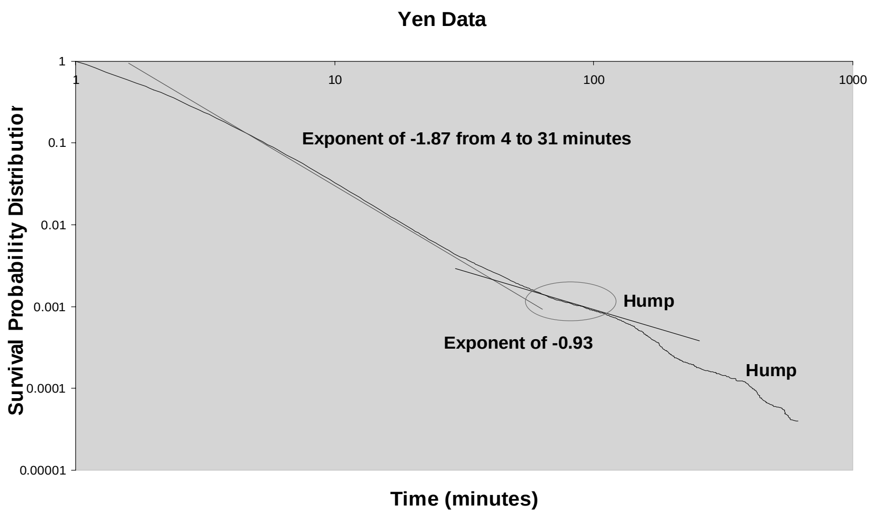

Fig. 2. Survival time probability distribution function for Yen Currency Market data between 1989 and 1998.

any exponential behavior has a relaxation time of less than a minute. This region appears to exhibit power law essentially over the complete range of data. But note that the power law now is greater than unity and the origin therefore must involve a mechanism outside that considered by Mainardi [3].

The shoulders do exhibit a very small region showing a power law on the boundary of the values admitted for β in (4). In fact, β ranges here between 0.9 and 1.1. However, the overall the distribution appears to be quite dissimilar to a Mittag Leffler function.

5 Conclusions

Continuous time random walk models, with a power law memory function in the waiting time dynamic, seem to reproduce the Survival time probability distributions for Irish stock market of 19th century and it would be interesting to see whether this holds for other early markets. However in their simple form, they do not appear to replicate the more complex features of modern currency markets. Again it would be interesting to explore this theory in the context of other contemporary assets.

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